PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as U=3PV+4.

1) either monoatomic or diatomic **2)** diatomic only

3) monoatomic only **4)** polyatomic only

Key: 4

Solution:

U =3nRT +4 $\frac{1}{n}\frac{dU}{dT} = 3R$ $C_v = 3R$ Polyatomic

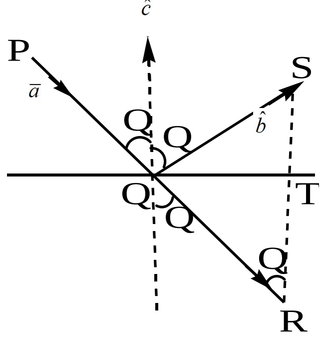
02. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

1) $\vec{b} = \vec{a} - 2(\vec{a}.\vec{c})\vec{c}$ **2)** $\vec{b} = \vec{a} - \vec{c}$ **3)** $\vec{b} = 2\vec{a} + \vec{c}$ **4)** $\vec{b} = \vec{a} + 2\vec{c}$

Key: 1

Solution:

$\overline{QS} = \overline{QR} + \overline{RS}$



$$\left(QS\right)\hat{b} = (QR)\overline{a} + \left(RS\right)\hat{c}$$

$$\hat{b} = \hat{a} + \frac{RS}{QS}.\hat{C}$$

$$\hat{b} = \hat{a} + 2\frac{RT}{QS}.\hat{C}$$

$$\overline{b} = \hat{a} - 2(\hat{a}.\hat{c})\hat{c}$$

03. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance ‘h’, the square of angular velocity of wheel will be:

1) $\frac{2mgh}{1+mr^2}$ **2)** $2gh$ **3)** $\frac{2mgh}{1+2mr^2}$ **4)** $\frac{2gh}{1+mr^2}$

Key: 4

Solution: Decrease In P.E of block = Increase in K.E of wheel + block

mgh = $\frac{1}{2}mv^2 + \frac{1}{2}IW^2$ where v = rw

mgh = $\frac{1}{2}mr^2w^2 + \frac{1}{2}IW^2$

$$W^2 = \frac{2mgh}{I + mr^2}$$

04. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be :

1) $\frac{a_1}{a_2}$ **2)** $\frac{a_2}{a_1}$ **3)** $\frac{a_1 + a_2}{a_1}$ **4)** $\frac{a_1 + a_2}{a_2}$

Key: 2

Solution:

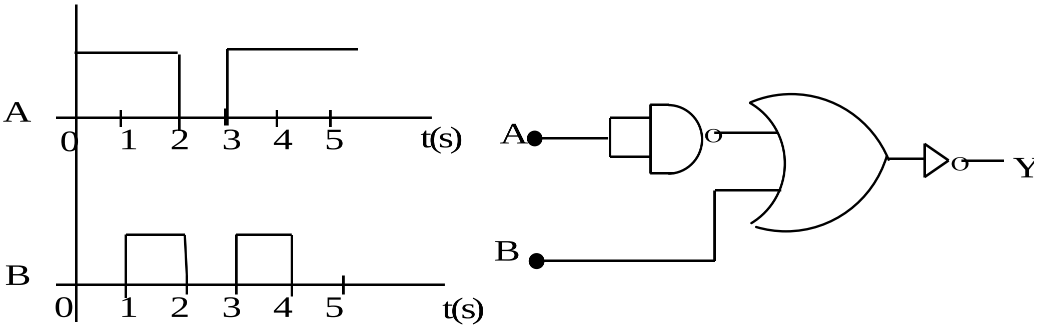
V = a_1t_1

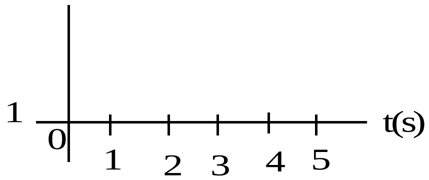
V = a_2t_2

$$a_1t_1 = a_2t_2$$

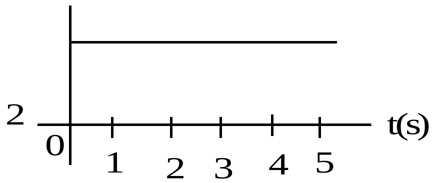
$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

05. Draw the output signal Y in the given combination of gates.





1)



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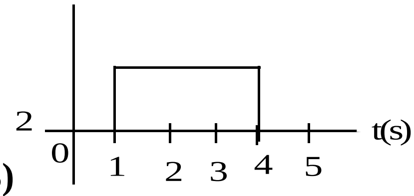
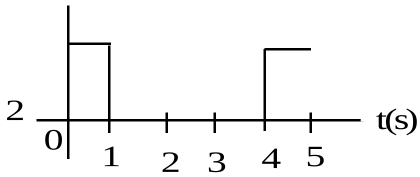


3)



4)

Key: 4

Solution:

A	B	Y = $\overline{\bar{A}+B}$
1	0	1
1	1	0
0	0	
1	1	0
1	0	1

06. Given below are two statements : one is labeled as Assertion A and the order is labeled as Reason R. Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image. Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large anlge. In the light of the above statements, choose the most appropriate answer from the options given below :

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is Not the correct explanation of A

3) A is false but R is true

4) A is true but R is false

Key:3

Solution:

Conceptual

07. A radioactive sample is undergoing α decay. At any time $t_{1,}$ its activity is A and another time t_2 , the activity is $\frac{A}{5}$. What is the average life time for the sample?

1) $\frac{t_2 - t_1}{In5}$ **2)** $\frac{In5}{t_2 - t_1}$ **3)** $\frac{t_1 - t_2}{In5}$ **4)** $\frac{In(t_2 + t_1)}{2}$

Key:1

Solution:

$$\frac{A}{5} = A.e^{-\lambda(t_2 - t_1)}$$

$$ln5 = \lambda(t_2 - t_1)$$

$$\lambda = \frac{\ln 5}{t_2 - t_1} \qquad T = \frac{1}{\lambda} = \frac{t_2 - t_1}{\ln 5}$$

08. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340Hz. When fork A is filed, the beat frequency decreases to 2beat/s. What is the frequency of fork A ?

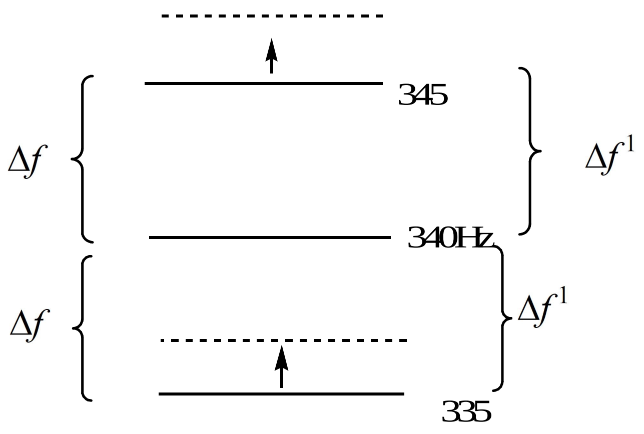
1) 338 Hz **2)** 335 Hz **3)** 342 Hz **4)** 345 Hz

Key:2

Solution:

$\Delta f = 5$

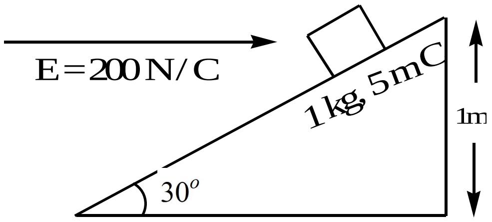
$\Delta f^1 = 2$



$$\therefore f_A = 335 Hz$$

9. An inclined plane making an angle of 30^0 with the horizontal is placed in a uniform horizontal electric field $200\frac{N}{C}$ as shown in the figure. A body of mass 1kg and charge 5mC is allowed to slide down from rest at a height of 1 m. If the coefficient of friction is 0.2 . Find the time taken by the body to reach the bottom.

$\left[g=9.8m/s^2;\sin 30^o=\frac{1}{2};\cos 30^o=\frac{\sqrt{3}}{2}\right]$



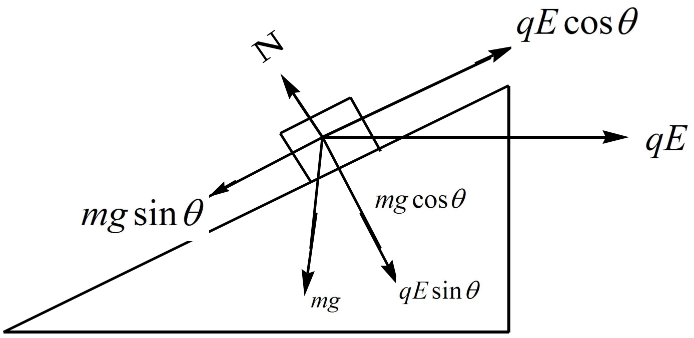
1) 0.92 s **2)** 1.3 s **3)** 2.3 s **4)** 0.46 s

Key:2

Solution:

$N=mg\cos\theta+qE\sin\theta$

$$a=\frac{mg\sin\theta-[qE\cos\theta+\mu N]}{m}$$



$ma=mg\sin\theta-[qE\cos\theta+\mu(mg\cos\theta+qE\sin\theta)]$

$a=g\sin\theta-\left[\frac{qE\cos\theta}{m}+\mu\left(q\cos\theta+\frac{qE\sin\theta}{m}\right)\right]$

$a=9.8\times\frac{1}{2}-\left[\frac{1\times\frac{\sqrt{3}}{2}}{1}+\frac{1}{5}\left(4.9\times\frac{\sqrt{3}}{2}+1\times\frac{1}{2}\right)\right]$

$=4.9-\frac{\sqrt{3}}{2}-\left(\frac{4.9\times\sqrt{3}}{10}-\frac{1}{10}\right)$ $=4.9\left[1-\frac{\sqrt{3}}{10}\right]-\left(\frac{5\sqrt{3}+1}{10}\right)$ $t=\sqrt{\frac{21}{a}}\approx 1.31s$

10. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where C / V= λ ?

1) $\left[M^{-2}L^{-4}I^{3}T^{7}\right]$ **2)** $\left[M^{-1}L^{-3}I^{-2}T^{-7}\right]$ **3)** $\left[M^{-3}L^{-4}I^{3}T^{7}\right]$ **4)** $\left[M^{-2}L^{-3}I^{2}T^{6}\right]$

Key:1

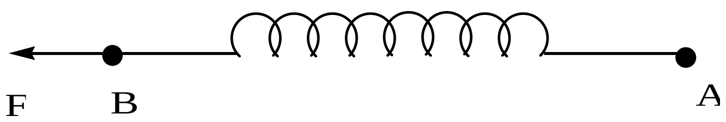
Solution:

P = Vi $\qquad U=\dfrac{9^2}{2c}$

V = $\dfrac{M^1L^2T^{-3}}{I^1}$ $C=\dfrac{I^2T^2}{M^1L^{-2}T^{-2}}$ $\qquad =M^1L^2T^{-3}I^{-1}$ $\qquad =M^{-1}L^{-2}T^4I^2$

$\dfrac{C}{V}=\dfrac{M^{-1}L^{-2}T^4I^2}{M^1L^2T^{-3}I^{-1}}$ $\qquad =M^{-2}L^{-4}T^7I^3$

11. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B will be:



1) $\dfrac{MF}{F+Ma}$ **2)** $\dfrac{F+Ma}{M}$ **3)** $\dfrac{Ma-F}{M}$ **4)** $\dfrac{F-Ma}{M}$

Key:4

Solution:

Gsytem = $\dfrac{F}{2M}$ for mass A

$G_A=\dfrac{Kx}{m}$

$a_B=\dfrac{F-Kx}{m}$ $\qquad a_B=\dfrac{F-Ma}{m}$

12. A particle executes S.H.M. the graph of velocity as a function of displacement is :

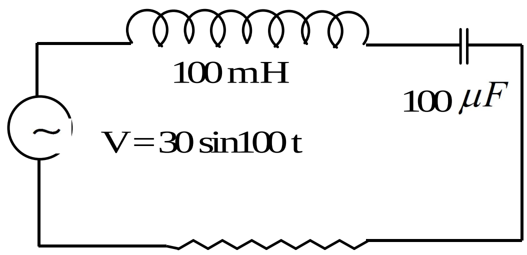
1) an ellipse **2)** a circle **3)** a helix **4)** a parabola

Key:1

Solution:

$$V = W\sqrt{A^2 - x^2} \qquad \frac{V^2}{(WA)^2} + \frac{x^2}{w^2} = 1 \qquad \therefore \text{ Graph is ellipse}$$

13. Find the peak current and resonant frequency of the following circuit (as shown in figure).



1) 0.2 A and 50 Hz **2)** 2A and 50 Hz

3) 0.2A and 100 Hz **4)** 2A and 100 Hz

Key:1

Solution:

$$X_c = WL = 100 \times 10^{-1} = 10Hz \qquad X_c = \frac{1}{WC} = \frac{1}{100 \times 10^{-4}} = 100Hz$$

$$Z = \sqrt{(X_C - X_L)^2 + R^2} = 150Hz$$

$$f_o = \frac{V_o}{Z} = \frac{30}{150} = 0.2A$$

$$W_o = 2\pi f_o = \frac{1}{\sqrt{LC}} \qquad f_o = \frac{1}{2\pi\sqrt{LC}} = 50Hz$$

14. Given below are two statements:

Statement I : A second's pendulum has a time period of 1 second

Statement II : It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options given

1) Both Statement I and Statement II are true

2) Both Statement I and Statement II are false

3) Statement I is true but Statement II is false

4) Statement I is false but Statement II is true

Key: 4

Solution:

Conceptual

15. An aeroplane, with its wings spread 10m, is flying at a speed of 180km/h in a horizontal direction. The total intensity of earth's field at that part is $2.5\times10^{-4}Wb/m^2$ and the angle of dip is 60^o. The emf induced between the tips of the plane wings will be ______

1) 108.25 mV **2)** 54.125Mv **3)** 88.37 mV **4)** 62.50mV

Key: 1

Solution:

$B_v = B_e \sin\delta \qquad = 2.5\times10^{-4}\times\frac{\sqrt{3}}{2}$

Emf induced = B_v lv =108.25mV

16. Given below are two statements:

Statement I **:** An electric dipole is placed at the centre of a hollow sphere . The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II **:** If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r(<R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, Choose the correct answer from the options given below :

1) Statement I is false but Statement II is true

2) Both Statement I and Statement II are true

3) Statement I is true but Statement II is false

4) Both Statement I and Statement II are false

Key: 4

Solution:

Conceptual

17. A wire of 1Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in resistance to the nearest integer is :

1) 76% **2)**12.5% **3)** 25% **4)** 56%

Key: 4

Solution:

On stretching,

$$R \alpha l^2$$

$$\frac{R_1}{R_2} = \frac{l^2}{(1.25l)^2} = \frac{1}{1.5625}$$

$$R_2 = R_1[1+0.56] \quad \therefore \frac{\Delta R}{R} + 100 = 56\%$$

18. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ the maximum height attained H are respectively given by :

1) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$ **2)** $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$ **3)** $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$ **4)** $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

Key:3

Solution:

$$y = \alpha x - \beta x^2$$

$$y = (\tan\theta)x - \left(\frac{g}{2x^2\cos^2\theta}\right)x^2$$

$$\tan\theta = \alpha \qquad \Rightarrow \theta = \tan^{-1}[\alpha]$$

$$\frac{\alpha^2}{4\beta} = \frac{\tan^2\theta}{4\left(\frac{g}{2u^2\cos^2\theta}\right)} = H$$

19. The length of metallic wire is l_1 when tension in it is T_1 . It is l_2 when the tension is T_2. The original length of the wire will be :

1) $\frac{l_1 + l_2}{2}$ **2)** $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$ **3)** $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$ **4)** $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$

Key:2

Solution:

For a wire meter some tension,

$$y = \frac{Tl}{Ae} \qquad \Rightarrow e \alpha T$$

$$\frac{l_1 - l_o}{l_2 - l_0} = \frac{T_1}{T_2}$$ Solving We get

$$l_o = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$$

20. The recoil speed of a hydrogen atom after it emits a photon in going from n=5 state to n=1 state will be :

1) 4.17 m/s **2)** 3.25 m/s **3)** 4.34 m/s **4)** 2.19 m/s

Key: 2

Solution:

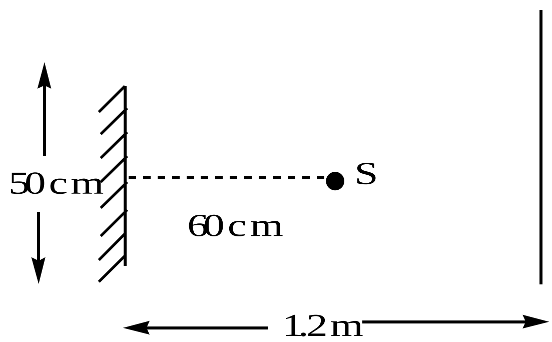
$$M V_{recoil} = \frac{h}{\lambda} = Rh\left[\frac{1}{1^2} - \frac{1}{5^2}\right]$$

$$V_{recoil} = \frac{Rh}{m}\left[\frac{24}{25}\right] \approx 3.85 ms^{-1}$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A point source of light S, placed at a distance 60cm infront of the centre of a plane mirror of width 50cm, hangs vertically on a wall. A man walks infront of the mirror along a line parallel to the mirror a distance 1.2m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is ____________cm.

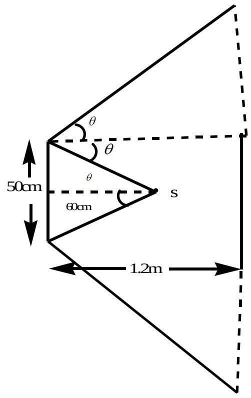


Key: 1501

Solution:

$$\tan\theta = \frac{25}{60} = \frac{4}{120}$$

$$\Rightarrow y = 501m$$



$$\therefore Total\,length = 3 \times 501m$$

$$= 1501m$$

22. 27 similar drops of mercury are maintained at 10V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is ________times that of a smaller drop.

Key: 9

Solution:

$R^3 = nr^3$ $\quad V_{big} = \frac{1}{4ner}.\frac{Q}{R} = \frac{K.[nq]}{n^{\frac{1}{3}}\gamma}$ $\quad V_{big} = n^{\frac{2}{3}}Vs = 9V_s$

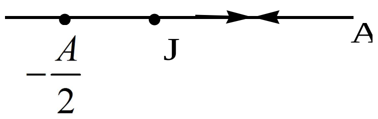
23. Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is ___________

Key: 7

Solution:

$\frac{5}{8}$ of oscillation means $\frac{5}{8}$(4A) = $\frac{5A}{2}$

$x = A\sin wt$ $\quad -\frac{A}{2} = A\sin wt$



$$\Rightarrow t = \frac{7T}{12} \quad \therefore \alpha = 7$$

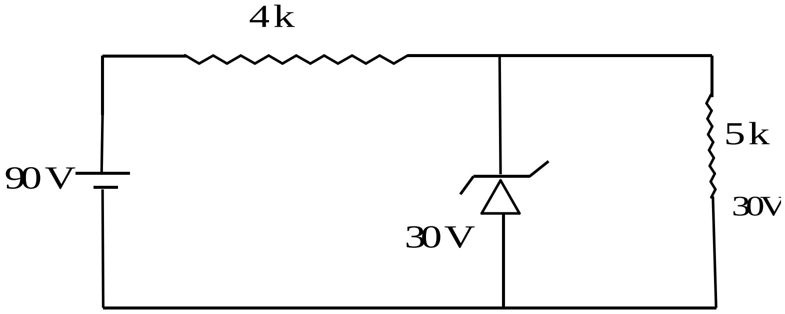
24. If the highest frequency modulating a carrier is 5kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are _________

Key: 18

Solution:

N = $\frac{90}{5} = 18$

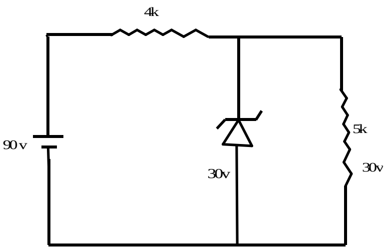
25. The zener diode has a V_z=30 V. The current passing through the diode for the following circuit is _________mA.



Key: 9

Solution:

$i = \frac{10}{4\times10^3} = 15mA$ $\quad\quad$ $i_l = \frac{3v}{5\times10^3} = 6ma$



$i_z = 9mA$

26. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it . The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$.The value of x is _____

[R=universal gas constant]

Key: 25

Solution:

Di atomic

$\gamma = \frac{7}{5}$ $\quad \Delta u = \frac{4u}{5} = 1\times\frac{5R}{2}\Delta T$ $\quad \frac{1}{n}.\frac{u}{\Delta T} = \frac{25R}{8}$ $\quad \boxed{x = 25}$

27. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases x : y .The value of x is___________

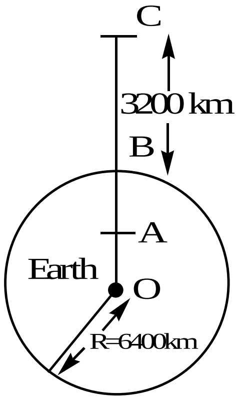
Key: 1

Solution:

$2W = W + \frac{1}{2}mV_1^2$ $\quad 10W = W + \frac{1}{2}mV_2^2$

$\frac{1}{9} = \frac{V_1^2}{V_2^2}$ $\quad \Rightarrow \frac{V_1}{V_2} = \frac{1}{3} = \frac{x}{y}$ $\quad \boxed{x = 1}$

28. In the reported figure of earth , the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth) . The value of OA : AB will be x : y. The value of x is __________



Key: 4

Solution:

$$g_n = g\frac{R^2}{(R+h)^2} = \frac{g.R^2}{\left(\frac{3R}{2}\right)^2} = \frac{4g}{9}$$

$$g_d = g\left[1-\frac{d}{R}\right] = \frac{4g}{9} \quad \frac{5}{9} = \frac{d}{R} \Rightarrow d = \frac{5R}{9} \qquad OA = x = \frac{4R}{9}$$

$$AB = \frac{5R}{9} \qquad OA : AB = 4:5 \qquad \boxed{x=4}$$

29. The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The workdone when temperature changes by 90 K will be xR The value of x is _________

[R =universal gas constant]

Key: 60

Solution: $PV = nRT \qquad V = K\left[\frac{pv}{nR}\right]^{\frac{2}{3}} \quad P^{\frac{2}{3}}V^{\frac{1}{3}} = constant \quad PV^{\frac{1}{2}} = constant \; W = \frac{RAT}{1-x} = 60R$

30. A particle executes S.H.M with amplitude 'a' and time period 'T'. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{xa}}{2}$.The value of x is ______

Key: $\Rightarrow \boxed{x = \frac{\sqrt{3}A}{2}}$

Solution:

$$V = W\sqrt{A^2 - x^2} \quad \text{where } V = \frac{AW}{2} \quad \frac{AW}{2} = W\sqrt{A^2-x^2} \quad \frac{A^2}{4} = A^2 - x^2$$

$$x^2 = \frac{3A^2}{4} \qquad \Rightarrow \boxed{x = \frac{\sqrt{3}A}{2}}$$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Given below are two statements: one labeled as **Assertion A** and the other is labeled as **Reason R**.

Assertion: In TII_3, isomorphous to CsI_3, the metal is present in +1 oxidation state.

Reason: TI metal has fourteen *f* electrons in its electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below.

1) Both A and R are correct and R is the correct explanation of A

2) Both A and R are correct and R is NOT the correct explanation of A

3) A is correct but R is not correct

4) A is not correct but R is correct

Key:2

Solution: 2

32. Ceric ammonium nitrate and $CHCl_3 / alc.KOH$ are used for the identification of functional groups present in ________ and ________ respectively.

1) amine, alcohol **2)** amine, phenol **3)** alcohol, phenol **4)** alcohol, amine

Key:4

Solution:

i) Ceric ammonium nitrate (CAN)

$$\text{alcohol} \xrightarrow[(mild)]{(O)} \text{aldehyde}$$

$$R-CH_2-OH \longrightarrow R-CHO$$

ii) Carbyl amine reaction.

33. The correct order of electron gain enthalpy is:

1) $S > O > Se > Te$ **2)** $Te > Se > S > O$

3) $S > Se > Te > O$ **4)** $O > S > Se > Te$

Key:3

Solution:

$$\boxed{O < Te < Se < S}$$

34. Calgon is used for water treatment. Which of the following statements is NOT true and Calgon?

1) Calgon contains the 2nd most abundant element by weight in the Earth's crust.

2) It doesnot remove Ca^{2+} ion by precipitation.

3) It is also known as Graham's salt

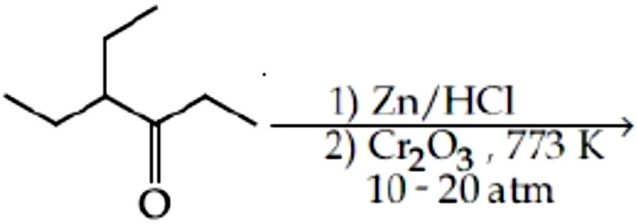
4) It is polymeric compound and is water soluble.

Key:1

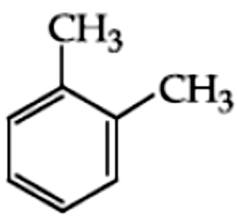
Solution:

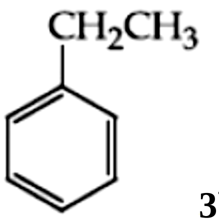
$$Na_2\left[Na_4P_6O_{18}\right] \rightarrow 2Na^+ + \left[Na_4P_6O_{18}\right]^{-2}$$

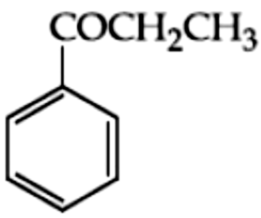
35.

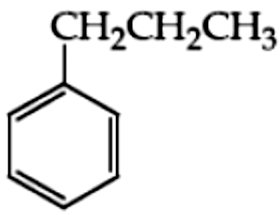


Considering the above reaction, the major product among the following is:

1) 

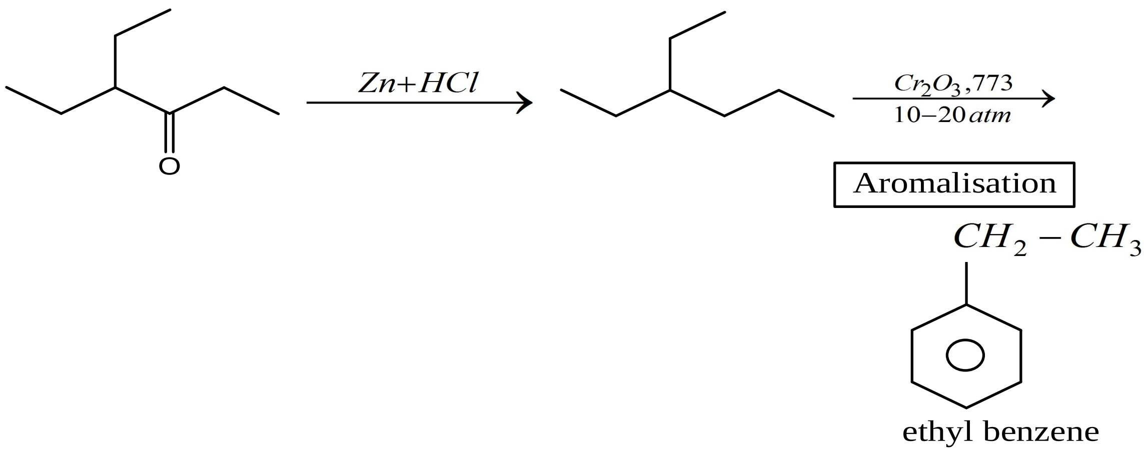
2) 

3) 

4) 

Key:2

Solution:



36. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is :

1) sometimes positive and sometimes negative

2) positive

3) negative

4) neutral

Key:2

Solution:

When $FeCl_3$ added to excess of hot water, a positively charged sol is formed due to adsorption of Fe^{3+} ions.

37. Seliwanoff test and Xanthroproteic test are used for the identification of ________ and respectively.

1) ketosese, proteins **2)** proteins, ketoses

3) ketoses, aldoses **4)** aldoses, ketoses

Key:1

Solution:

Seliwan off test $\rightarrow$ distinguish test for carbohydrates.

X anthoprotic test: $\rightarrow$ Distinguish test for proteins

38. Match List – I with List – II.

List – I (Molecule)	**List – II (Bond Order)**
a) Ne_2	i) 1
b) N_2	ii) 2
c) F_2	iii) 0
d) O_2	iv) 3

Choose the correct answer from the options given below:

1) $(a)\rightarrow(i),(b)\rightarrow(ii),c\rightarrow(iii),d\rightarrow(iv)$

2) $(a)\rightarrow(iv),(b)\rightarrow(iii),c\rightarrow(ii),d\rightarrow(i)$

3) $(a)\rightarrow(ii),(b)\rightarrow(i),c\rightarrow(iv),d\rightarrow(iii)$

4) $(a)\rightarrow(iii),(b)\rightarrow(iv),c\rightarrow(i),d\rightarrow(ii)$

Key:4

Solution:

$Ne_2 \to BO \to 0 \qquad N_2 \to BO \to 3 \qquad O_2 \to BO \to 2 \qquad F_2 \to BO \to 1$

39. A. Phenyl methanamine

B. N, N-Dimethylaniline

C. N-Methyl aniline

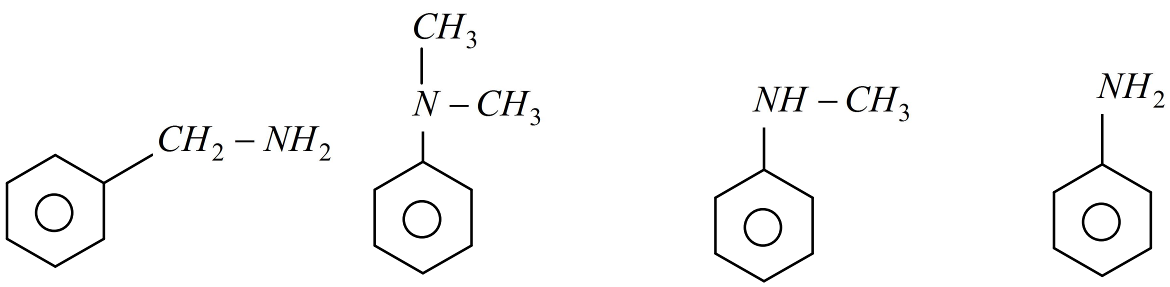
D. Benzenamine

Choose the correct order of basic nature of the above amines.

1) $D > C > B > A$ **2)** $A > B > C > D$ **3)** $A > C > B > D$ **4)** $D > B > C > A$

Key:2

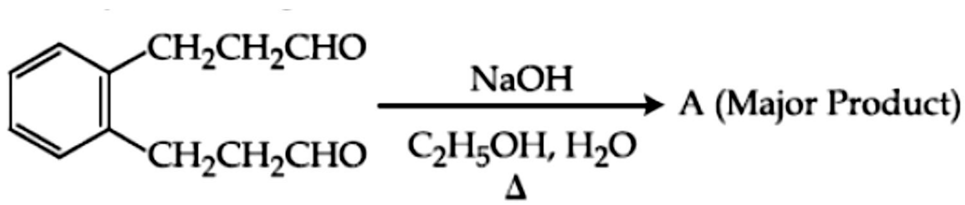
Solution:

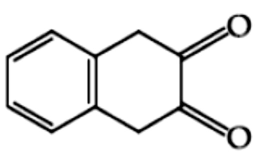


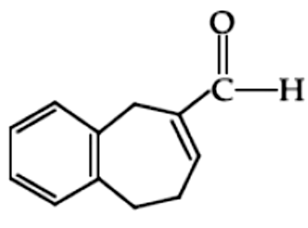
-1^o- (A) $\qquad -3^o-$ (B) $\qquad -2^o-$ (C) $\qquad$ (D)

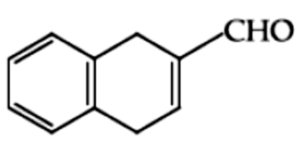
$\boxed{A > B > C > D}$

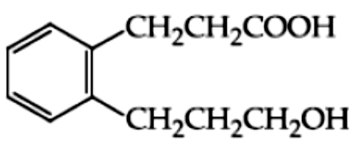
40. Identify A in the given chemical reaction



1) 

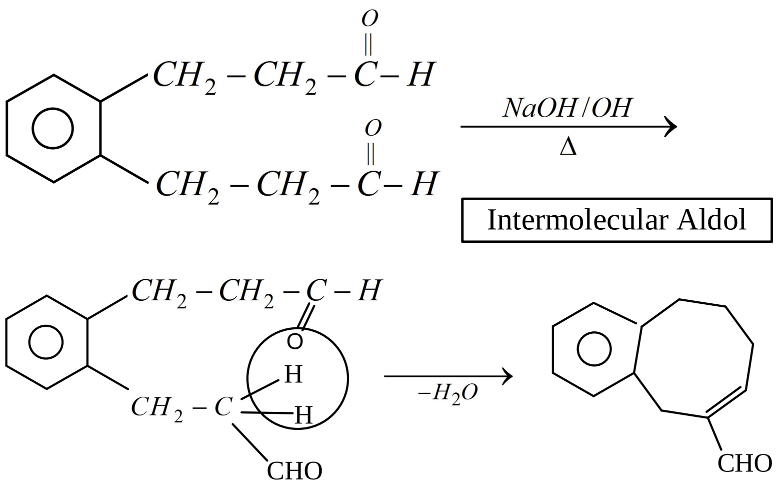
2) 

3) 

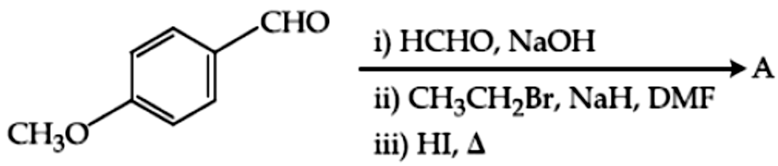
4) 

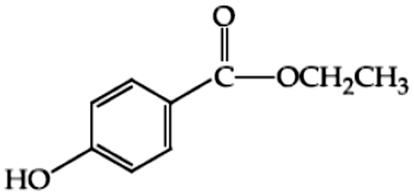
Key:2

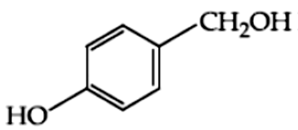
Solution:

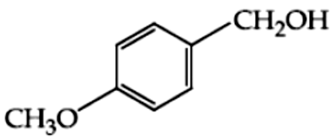


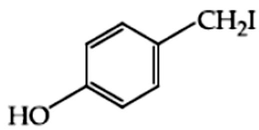
41. Identify A in the following chemical reaction.



1) 

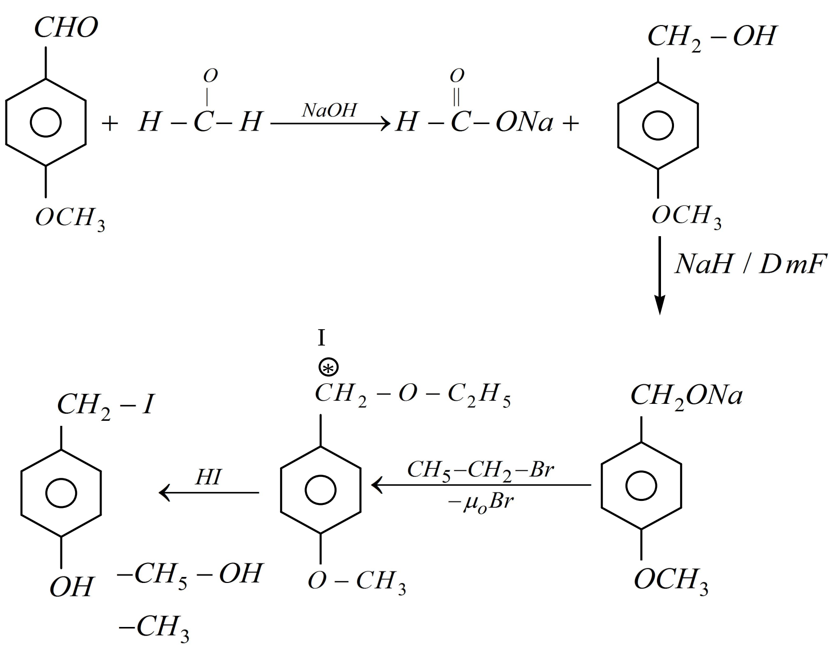
2) 

3) 

4) 

Key:4

Solution:



42. Which pair of oxides is acidic in nature?

1) B_2O_3, CaO **2)** B_2O_3, SiO_2 **3)** CaO, SiO_2 **4)** N_2O, BaO

Key:2

Solution:

Oxide		**Nature**
CaO	$\longrightarrow$	Basic
ZnO	$\longrightarrow$	Amphoteric
B_2O_3	$\longrightarrow$	acidic
SiO_2	$\longrightarrow$	acidic

43. In $\overset{1}{C}H_2 = \overset{2}{C} = \overset{3}{C}H - \overset{4}{C}H_3$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively, are:

1) sp^2, sp^3, sp^2, sp^3 **2)** sp^2, sp, sp^2, sp^3

3) sp^2, sp^2, sp^2, sp^3 **4)** sp^3, sp, sp^3, sp^3

Key:2

Solution:

$$\underset{sp^2}{\underset{\downarrow}{CH_2}} = \underset{sp}{\underset{\downarrow}{C}} = \underset{sp^2}{\underset{\downarrow}{CH}} - \underset{sp^3}{\underset{\downarrow}{CH_3}}$$

44. Match List – I with List – II

List – I	**List – II**
a)	Siderite i) Cu
b) Calamine	ii) Ca
c) Malachite	iii) Fe
d) Cryolite	iv) Al
	v) Zn

Choose the correct answer from the options given below:

1) $(a) \to (i), (b) \to (ii), (c) \to (v), (d) \to (iii)$

2) $(a) \to (i), (b) \to (ii), (c) \to (iii), (d) \to (iv)$

3) $(a) \to (iii), (b) \to (i), (c) \to (v), (d) \to (ii)$

4) $(a) \to (iii), (b) \to (v), (c) \to (i), (d) \to (iv)$

Key:4

Solution:

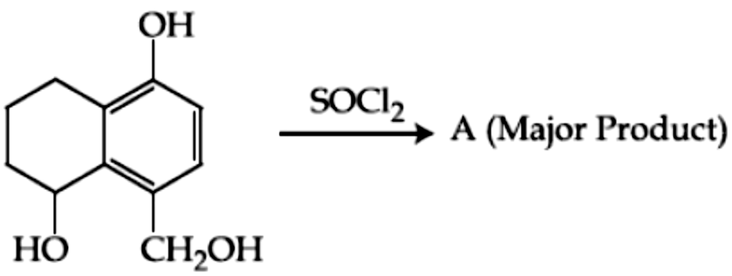
Siderite $\longrightarrow$ $\underline{Fe}CO_3$

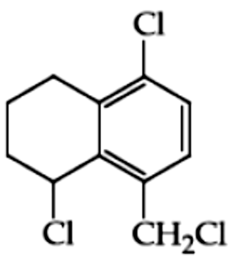
Calamine $\longrightarrow$ $\underline{Zn}CO_3$

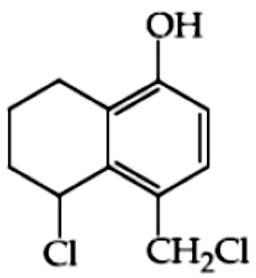
Malachite $\longrightarrow$ $\underline{Cu}(OH)_2.CuCO_3$

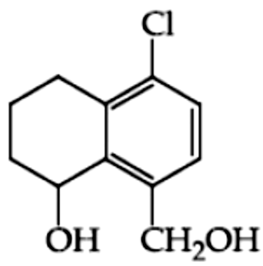
Cryolite $\longrightarrow$ $Na_3\underline{Al}F_6$

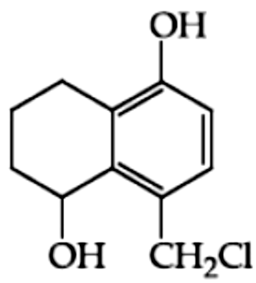
45. Identify A in the given reaction.



1) 

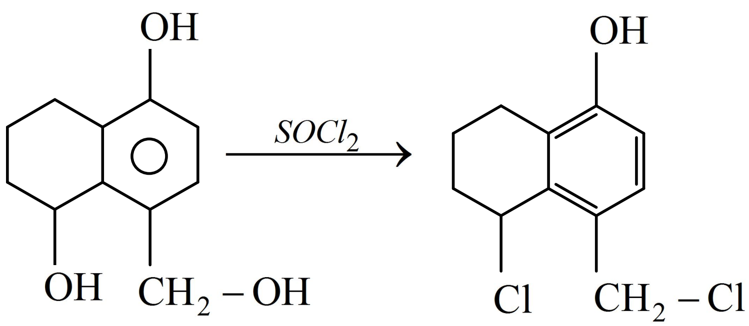
2) 

3) 

4) 

Key:2

Solution:



46. 2, 4 – DNP test can be used to identify:

1) aldehyde **2)** halogens **3)** amine **4)** ether

Key:1

Solution:

2, D, DNP test is the test for carbonyl compounds (or) Aldehydes and ketones.

47. Match List – I with List – II.

List – I	List – II
a)Sucrose	i) $\beta-D-$Galactose and $\beta-D-$Glucose
b)Lactose	ii) $\alpha-D-$Glucose and $\beta-D-$Fructose
c)Maltose	iii) $\alpha-D-$Glucose and $\alpha-D-$Glucose

Choose the correct answer from the options given below:

1) $(a)\to(i),(b)\to(iii),(c)\to(ii)$ **2)** $(a)\to(iii),(b)\to(i),(c)\to(ii)$

3) $(a)\to(ii),(b)\to(i),(c)\to(iii)$ **4)** $(a)\to(iii),(b)\to(ii),(c)\to(i)$

Key:3

Solution:

a) Sucrose $\to\alpha-$ Glucose & β - fructose

b) Lactose $\to\beta$ - Galactose & $\beta-$ Glucose

c) Maltose $\to$ $\alpha-$ Glucose & $\alpha-$ Glucose

48. Match List – I with List – II.

List – I	List – II
a)Sodium Carbonate	i) Deacon
b)Titanium	ii) Castner-Kellner
c)Chlorine	iii) van-Arkel
d) Sodium hydroxide	iv) Solvay

Choose the correct answer from the options given below:

1) $(a)\to(iii),(b)\to(ii),(c)\to(i),(d)\to(iv)$

2) $(a)\to(iv),(b)\to(i),(c)\to(ii),(d)\to(iii)$

3) $(a)\to(i),(b)\to(iii),(c)\to(iv),(d)\to(ii)$

4) $(a)\to(iv),(b)\to(iii),(c)\to(i),(d)\to(ii)$

Key:4

Solution:

a) Na_2CO_3 $\to$ Solvay process

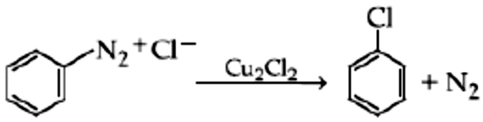
b) Ti $\to$van – Arkel process

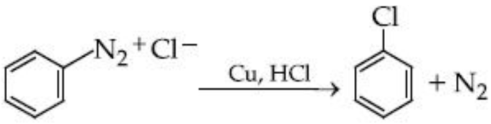
c) Cl_2 $\to$ Deacons process

d) $NaOH$ $\to$ Castner – Kellner process

49. Match List-I with List-II

List-I | **List-II**

a) 
i) Wurtz reaction

b) 
ii) Sandmeyer reaction

c) $2CH_3CH_2Cl + 2Na \xrightarrow{Ether} C_2H_5 - C_2H_5 + 2NaCl$
iii) Fitting reaction

d) $2C_6H_5Cl + 2Na \xrightarrow{Ether} C_6H_5 - C_6H_5 + 2NaCl$
iv) Gattermann reaction

Choose the correct answer from the options given below:

1) $(a) \to (iii), (b) \to (iv), (c) \to (i), (d) \to (ii)$

2) $(a) \to (ii), (b) \to (i), (c) \to (iv), (d) \to (iii)$

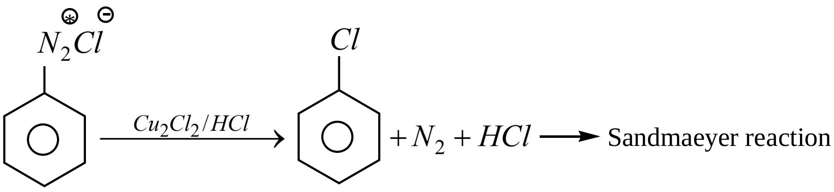
3) $(a) \to (iii), (b) \to (i), (c) \to (iv), (d) \to (ii)$

4) $(a) \to (ii), (b) \to (iv), (c) \to (i), (d) \to (iii)$

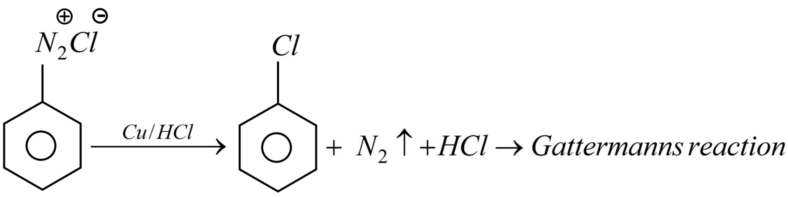
Key:4

Solution:

a)



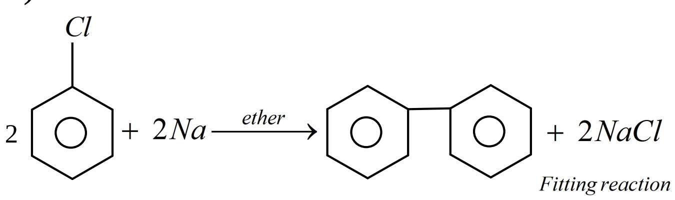
b)



c) $2CH_3 - CH_2 - Cl + 2Na \xrightarrow{ether} C_2H_5 + C_2H_5 + 2NaCl$

Wurtz reaction

d)



50. Which of the following forms of hydrogen emits low energy β^{-} particles?

1) Tritium $_1^3\text{H}$ **2)** Protium $_1^3\text{H}$ **3)** Deuterium $_1^2\text{H}$ **4)** Proton H^{+}

Key:1

Solution:

Conceptual

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. The number of octahedral voids per lattice site in a lattice is ________ (Rounded off to the nearest integer)

Key:1

Solution:

No. of lattice points/units cell of FCC lattice = 4
No. of octahedral voids = 4
$\therefore$ No. of $OV_s\,/\,lattice$ point = 1

52. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^{-} to “A” the oxidation state of sulphur in ‘A’ is _______

Key:6

Solution:

$$S_2O_3^{-2} + MnO_4^{-} \longrightarrow MnO_2 + \underset{(A)}{\overset{+6}{S}O_4^{-2}}$$

53. If the activation energy of a reaction is $80.9\,kJ\,mol^{-1}$, the fraction of molecules at 700 K, having enough energy to react to form products is e^{-x}. The value of x is ________. (Rounded off to the nearest integer) [Use $R = 8.31\,J\,K^{-1}mol^{-1}$]

Key:14

Solution:

Fraction of reactant molecules having energy equal to or greater than threshold $= e^{-Ea/RT}$

$= e^{-\frac{80900}{8.31\times 700}}$

$= e^{-14}$

54. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^{+} per mL is _______ g. (Rounded off to the nearest integer)

[Given: Atomic weight in $g\,mol^{-1}$ $- Na:23; N:14; O:16$]

Key:13

Solution:

$NaNO_3 \rightarrow Na^{+} + NO_3^{-}$

1mole

$85g \longrightarrow 23g$

$85mg \longrightarrow 23mg$

$? \longrightarrow 70mg$

$= \dfrac{70 \times 85}{23} mg$

Mass of $NaNO_3$ required for 1 mL of solution $= \dfrac{70 \times 85}{23} mg$

For 50 mL of solution, mass of $NaNO_3 = \dfrac{70 \times 85}{23} \times 50$

$= 12934\, mg$

$= 12.93\, g$

$\approx 13\, g$

55. The pH of ammonium phosphate solution, if pk_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is ________.

Key: 7.24

Solution:

For aq. Solution of salt of weak acid with weak base,

$$pH = \frac{1}{2}\left(pk_w + pk_a - pk_b\right)$$

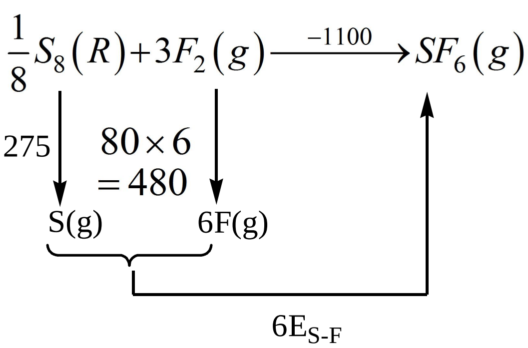
$$= 7 + \frac{5.23}{2} - \frac{4.75}{2} \quad = 7 + 2.62 - 2.38 \quad = 7.24$$

56. The average $S-F$ bond energy in $kJ\,mol^{-1}$ of SF_6 ________. (Rounded off to the nearest integer).

[Given: The values of standard enthalpy of formation of $SF_6(g), S(g)$ and $F(g)$ are –1100, 275 and $80\,kJ\,mol^{-1}$ respectively.]

Key: 309.17

Solution:



Hess' law:

$$-1100 = 275 + 480 + 6E_{S-F}$$

$$6E_{S-F} = -1855$$

$$E_{S-F} = -309.17\, kg\ /\ mol$$

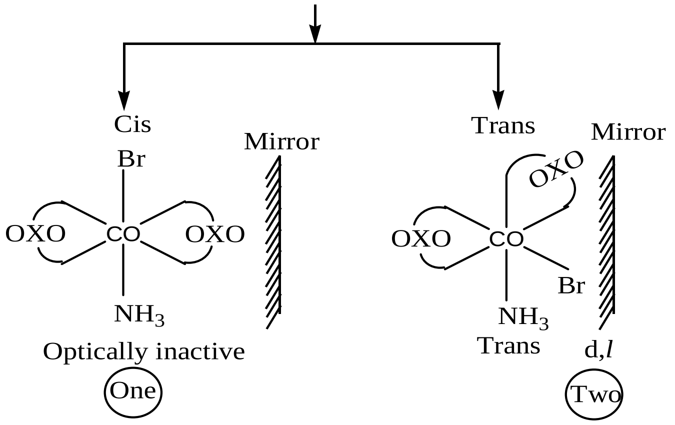
$$= +309.17\, kJ\ /\ mol$$

57. The number of stereoisomers possible for $\left[Co\left(OX\right)_2\left(Br\right)\left(NH_3\right)\right]^{2-}$ is _______ [OX = oxalate].

Key: 3

Solution:

$\left[CO(OXO)_2\, Br\left(NH_3\right)\right]^{-2} \rightarrow \boxed{Total(3)}$



58. When 12.2 g of benzoic acid is dissolved in 100 g of water, the freezing point solution was found to be $-0.93^oC\left(K_f\left(H_2O\right) = 1.86\, K\, kg\, mol^{-1}\right)$. The number (n) of benzoic acid molecules associated (assuming 100% association) is ________ .

Key: 2

Solution:

$$\Delta T_f = ik_f m$$

$$0.93 = i \times 1.86 \times \frac{12.2 \times 1000}{122 \times 100}$$

$$i = 0.5$$

$$\alpha = \frac{1-i}{1-\frac{1}{n}}$$

$$1 = \frac{1-0.5}{1-1/n}$$

$$1 - \frac{1}{n} = 1 - 0.5$$

$$n = 2$$

59. Emf of the following cell at 298 K in V is $x \times 10^{-2}$.

$$Zn | Zn^{2+}(0.1M) || Ag^{+}(0.01M) | Ag$$

The value of *x* is ________. (Round off to the nearest integer)

[Give: $E^{\theta}_{Zn^{2+}/Zn} = -0.76V; E^{\theta}_{Ag^{+}/Ag} = +0.80V; \frac{2.303RT}{F} = 0.059$]

Key: 153

Solution:

$$Zn | Zn^{2+}(0.1M) || Ag^{+}(0.01M) | Ag$$

$$E^{o}_{cell} = E^{o}_{C} - E^{o}_{A} = 0.80 - (-0.76) = +1.56V$$

Cell $lxn: Zn(s) + 2Ag^{+}(aq) \rightarrow \overset{2+}{Zn}(aq) + 2Ag(s)(n=2)$

$$Q_R = \frac{[Zn^{2+}]}{[Ag^{+}]^2}$$

$$E_{cell} = E^{o}_{call} - \frac{0.059}{n}\log Q_R$$

$$= 1.56 - \frac{0.059}{2}\log\frac{0.1}{0.01}$$

$$= 1.56 - 0.0295 = 1.5305V$$

$$= 153.05 \times 10^{-2} V$$

$$= 153 \times 10^{-2} V$$

60. A ball weighing 10g is moving with a velocity of $90 ms^{-1}$. If the uncertainty in its velocity is 5%, then the uncertainty in its position is ________ $\times 10^{-33} m$. (Rounded off to the nearest integer)

[Given: $h = 6.63 \times 10^{-34} Js$]

Key: 1.17

Solution:

$$\Delta x.\Delta v = \frac{h}{4\pi m}$$

$$= \frac{6.626 \times 10^{-34} J\sec}{4 \times 3.14 \times 10 \times 10^{-3} \times 4.5} \quad \frac{5}{100} \times 90 = 4.5$$

$$= \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 10 \times 10^{-3} \times 4.5}$$

$$= \frac{6.626 \times 10^{-34}}{565.2 \times 10^{-3}}$$

$$= 0.01172 \times 10^{-31}$$

$$= 1.17 \times 10^{-2} \times 10^{-31} \qquad = 1.17 \times 10^{-33} m$$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. Let L be a line obtained from the intersection of two planes $x+2y+z=6$ and $y+2z=4$ If point P (α,β,γ) is the foot of perpendicular from (3,2,**1)** on L, then the value of $21(\alpha+\beta+\gamma)$ equals

1) 68 **2)** 102 **3)** 142 **4)** 136

Key:2

Solution:

$x+2t+z=6, y+z=4$

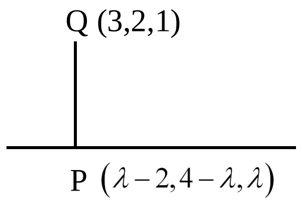
$x+2y+z-6=0, y+z-4=0$

Solving the line is

$$\frac{x+2}{1}=\frac{y-4}{-1}=\frac{z}{1}$$

Any point on this line is $(\lambda-2,4-\lambda,\lambda)$

Foot of $\perp r$ from $(3,2,1)$ to the line is $p(\alpha,\beta,\gamma)$



d.rs of $PQ=(\lambda-5,2-\lambda,\lambda-1)$

$$\therefore(\lambda-5)5+(2-\lambda)(-1)+1(\lambda-1)=0\Rightarrow 3\lambda-8=0\Rightarrow\lambda=\frac{8}{3}$$

$$\therefore p=\left(\frac{2}{3},\frac{4}{3},\frac{8}{3}\right)=(\alpha,\beta,\gamma)\ 21(\alpha+\beta+\gamma)=21\left(\frac{14}{3}\right)=98$$

62. If the mirror image of the point(1,3,5) with respect to the plane $4x-5y+2z=8$ is (α,β,γ) then $5(\alpha+\beta+\gamma)$ equals:

1) 39 **2)** 41 **3)** 43 **4)** 47

Key:4

Solution:

$$\frac{\alpha-1}{4}=\frac{\beta-3}{-5}=\frac{\gamma-5}{2}=\frac{-2[4-15+10-8]}{16+25+4}=\frac{2}{5}$$

$$\therefore \alpha=\frac{13}{5},\beta=1,\gamma=\frac{29}{5}$$

$$5(\alpha+\beta+\gamma)=47$$

63. Let $A(1,4)$ and $B(1,-5)$ be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points, P,A and B lie on:

1) a hyperbola **2)** a parabola **3)** a straight line **4)** an ellipse

Key:3

Solution:

$$P=(1+\cos\theta,1+\sin\theta),A=(1,4),B(1,-5)$$

$$PA^2+PB^2=\cos^2\theta+(\sin\theta-3)^2+\cos^2\theta+(\sin\theta+6)^2$$

$$=10-6\sin\theta+37+12\sin\theta$$

$$=47+12\sin\theta-6\sin\theta=47+6\sin\theta$$

$$\left(PA^2+PB^2\right)_{\max}=(47+6\sin\theta)_{\max}\Rightarrow\theta=\frac{\pi}{2} \qquad \therefore p=(1,2)$$

$\therefore p,A,B$ lie on line $x=1$

64. Let $f(x)=\sin^{-1}x$ and $g(x)=\dfrac{x^2-x-2}{2x^2-x-6}$. If $g(2)=\lim\limits_{x\to 2}g(x)$, then the domain of the function fog is

1) $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$ **2)** $(-\infty,-2]\cup[-1,\infty)$

3) $(-\infty,-1]\cup[2,\infty)$ **4)** $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

Key:1

Solution:

$$f(x)=\sin^{-1}x,g(x)=\frac{x^2-x-2}{2x^2-x-6},x\neq 2$$

$$g(2)=\lim_{x\to 2}g(x)=\frac{3}{7}$$

$$(fog)(x)=f(g(x))=\sin^{-1}(g,(x))\Rightarrow\left|\frac{x^2-x-2}{2x^2-x-6}\right|\le 1$$

$$\Rightarrow\left|\frac{(x-2)(x+1)}{(x-2)(2x+3)}\right|\le 1\Rightarrow\left|\frac{x+1}{2x+3}\right|\le 1\Rightarrow -1\le\frac{x+1}{2x+3}\le 1$$

$$\Rightarrow\frac{3x+4}{2x+3}\ge 0 \text{ and } \frac{-2-x}{2x+3}\le 0\Rightarrow x\in\left(-\infty,-\frac{3}{2}\right)\cup\left[\frac{-4}{3},\infty\right)$$

$$\text{and } x\in(-\infty,-2]\cup(-3/2,\infty)\Rightarrow x\in(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right]$$

65. Consider the following system of equations:

$x+2y-3z=a$

$2x+6y-11z=b$

$x-2y+7z=c$

Where a,b and c are real constants. Then the system of equations:

1) has a unique solution for all a,b and c

2) has no solution for all a,b and c

3) has infinite number of solutions when $5a=2b+c$

4) has a unique solution when $5a=2b+c$

Key:3

Solution:

$x+2y-3z=a \quad 2x+6y-11z=b \quad x-2y+7z=c$

$$\Delta=\begin{vmatrix}1 & 2 & -3\\ 2 & 6 & -11\\ 1 & -2 & 7\end{vmatrix}=0 \qquad \Delta_1=\begin{vmatrix}a & 2 & -3\\ b & 6 & -11\\ c & -2 & 7\end{vmatrix}=4(5a-2b-c)$$

$$\Delta_2=\begin{vmatrix}1 & a & -3\\ 2 & b & -11\\ 1 & c & 7\end{vmatrix}=-5(5a-2b-c) \quad \Delta_3=\begin{vmatrix}1 & 2 & a\\ 2 & 6 & b\\ 1 & -2 & c\end{vmatrix}=-2(5a-2b-c)$$

$\therefore$ If $5a=2b+c\Rightarrow\Delta_1=\Delta_2=\Delta_3=0\Rightarrow$ Infinitely many solutions

66. If vectors $\overrightarrow{a_1}=x\hat{i}-\hat{j}+\hat{k}$ and $\overrightarrow{a_2}=\hat{i}-y\hat{j}+z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i}+y\hat{j}+z\hat{k}$ is :

1) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$ **2)** $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ **3)** $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$ **4)** $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

Key:1

Solution:

$\overrightarrow{a_i} = x\bar{i} - \bar{j} + \bar{k}, a_2 = \bar{i} + y\bar{j} + z\bar{k}$ are collinear $\qquad \frac{x}{1} = \frac{-1}{y} = \frac{1}{z} = \lambda$ say

Required $= \frac{\lambda\bar{i} - \frac{1}{\lambda}\bar{j} + \frac{1}{\lambda}E}{\sqrt{\lambda^2 + \frac{1}{\lambda^2} + \frac{1}{\lambda^2}}}$ $\qquad$ for $\lambda = 1 = \frac{\bar{i} - \bar{j} + \bar{k}}{\sqrt{3}}$

67. Let $F_1(A,B,C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A,B) = (A \vee B) \vee (B \to \sim A)$ be two logical expressions. Then:

1) F_1 is not a tautology but F_2 is a tautology

2) Both F_1 and F_2 are not tautologies

3) F_1 and F_2 both are tautologies

4) F_1 is a tautology but F_2 is not a tautology

Key:1

Solution:

$F_1(A,B,C) = (\sim A \vee B) \vee (\sim A) \vee (\sim c \wedge (A \vee B))$

$F_2(A,B) = (A \vee B) \vee (B \to \sim A)$

F_1 not tautology; F_2 is tautology

A	B	C	$\sim A$	$\sim C$	$A \vee B$	$\sim A \vee B$	$\sim C \wedge (A \vee B)$	$(\sim A \vee B)$ $\vee(\sim C(A \vee B)$ $\vee(\sim A)$
T	T	T	F	F	T	T	F	T
T	F	F	F	T	T	F	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F	F
F	T	T	T	F	T	T	F	T
F	F	F	T	T	F	T	F	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	F	T	F	T

Truth table for F_2

A	B	$A\vee b$	$\sim A$	$B\to\sim A$	$(A\vee B)\vee(B\to\sim A)$
T	T	T	F	F	F
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	T	T

68. Let slope of the tangent line to a curve at any point $P(x,y)$ be given by $\frac{xy^2+y}{x}$. If the curve intersects the line $x+2y=4$ at x = -2, then the value of y, for which the point $(3,y)$ lies on the curve, is :

1) $-\frac{18}{11}$ **2)** $-\frac{4}{3}$ **3)** $\frac{18}{35}$ **4)** $-\frac{18}{19}$

Key:4

Solution:

$$\frac{dy}{dx}=\frac{xy^2+y}{x} \qquad xdy-ydx=xy^2dx$$

$$\frac{ydx-xdy}{y^2}=-xdx\Rightarrow\int d\left(\frac{x}{y}\right)=fxdx\ \Rightarrow\frac{x}{y}=\frac{-x^2}{2}+c$$

$\downarrow$passes through $(-2,3)\Rightarrow\frac{-2}{3}=-2+c\Rightarrow c=\frac{4}{3}$

Curve is $\frac{x}{y}=\frac{-x^2}{2}+\frac{4}{3}$

$(3,y)$lies on it$\Rightarrow\frac{3}{y}=\frac{-9}{2}+\frac{4}{3}\Rightarrow\frac{3}{y}=\frac{-19}{6}\Rightarrow y=\frac{-18}{19}$

69. If $0<a,b<1$, and $\tan^{-1}a+\tan^{-1}b=\frac{\pi}{4}$, then the value of

$$(a+b)-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)-\left(\frac{a^4+b^4}{4}\right)+\ldots\text{is}$$

1) $\log_e\left(\frac{e}{2}\right)$ **2)** e **3)** $\log_e 2$ **4)** e^2-1

Key:3

Solution:

$$Tan^{-1}(a)+Tan^{-1}(b)=\frac{\pi}{4}\Rightarrow\frac{a+b}{1-ab}=1\Rightarrow ab+ab=1$$

$$=\left(a-\frac{a^2}{2}+\frac{a^3}{3}-\frac{a^4}{4}+....\infty\right)+\left(b-\frac{b^2}{2}+\frac{b^3}{3}-\frac{b^4}{4}+.....\infty\right)$$

$$=\ln(1+a)+\ln(1+b)$$

$$=\ln((1+a)(1+b))$$

$$=\ln(1+a+b+ab)=\ln 2$$

70. Let $f(x)=\int_0^x e^t f(t)dt+e^x$ be a differentiable function for all $x\in R$. Then $f(x)$ equals:

1) $e^{\left(e^x-1\right)}$ **2)** $2e^{\left(e^x-1\right)}-1$ **3)** $e^{e^x}-1$ **4)** $2e^{e^x}-1$

Key:2

Solution:

$$f(x)=\int_0^x e^t f(t)dx+e^x \qquad f'(x)=e^x f(x)+e^x$$

$$=e^x\left(f(x)+1\right)\Rightarrow\frac{f'(x)}{f(x)+1}=e^x\Rightarrow\int\frac{f'(x)}{f(x)+1}dx=\int e^x dx$$

$$\Rightarrow\ln(1+f(x))=e^x+c \text{ but } f(0)=1\therefore\ln 2=1+c$$

$$\ln(1+f(x))=e^x+\ln 2-1=\ln\left(2.e^{e^{x-1}}\right)$$

$$1+f(x)=2.e^{\left(e^x-1\right)} \quad f(x)=2e^{\left(e^x-1\right)}-1$$

71. Let $f:R\to R$ be defined as

$$f(x)=\begin{cases}2\sin\left(-\frac{\pi x}{2}\right), & if \quad x<-1\\ \left|ax^2+x+b\right|, & if \quad -1\le x\le 1\\ \sin(\pi x), & if \quad x>1\end{cases}$$

If $f(x)$ is continuous on R, then a+b equals:

1) -3 **2)** 3 **3)** 1 **4)** -1

Key:4

Solution:

$$f(x) = 2\sin\left(-\frac{\pi x}{2}\right), x < -1 = \left|ax^2 + x + b\right|, -1 \le x \le 1$$

$= \sin \pi x, x > 1$ Is continuous $\forall x \in R$

$$\Rightarrow f\left(-1^-\right) = f\left(-1^+\right) = f(-1), f\left(1^-\right) = \left(1^+\right) = f(1)$$

$$\Rightarrow 2 = |a+b-1|, |a+b+1| = 0 \quad a+b+1 = 0 \Rightarrow a+b = -1$$

72. For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to

1) -1 **2)** 0 **3)** 1 **4)** $\frac{1}{2}$

Key:4

Solution:

$$f(x) = \int_1^x \frac{\log_e t}{1+t} dt \quad f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{\log t}{1+t} dt$$

Put $t = \frac{1}{z} = \int_1^x \frac{-\ln z}{1+\frac{1}{z}} \left(\frac{-1}{z^2}\right) dz = \int_1^x \frac{\ln z}{z(1+z)} dz$

$$= \int_1^x \frac{\ln t}{t(1+t)} dt \therefore f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\ln t}{1+t} dt + \int_1^e \frac{\ln t}{t(1+t)} dt$$

$$= \int_1^e \frac{\ln t}{t} dt = \left(\frac{(\ln t)^2}{2}\right)_1^e = \frac{1}{2}$$

73. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}, y > z$. Then the number of odd divisors of n, including 1, is

1) 6 **2)** 11 **3)** 12 **4)** $6x$

Key:3

Solution:

$$y + z = 5, \frac{1}{y} + \frac{1}{z} = \frac{5}{6}, y > z \; N = 2^x 3^y 5^z$$

Solving $y = 3, z = 2$

No.of odd divisors of $N=(3+1)(2+1)=12$

74. Let A_1 be the area of the region bounded by the curves $y=\sin x, y=\cos x$ and $y-axis$ in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y=\sin x, y=\cos x, x-axis$ and $x=\frac{\pi}{2}$ in the first quadrant. Then

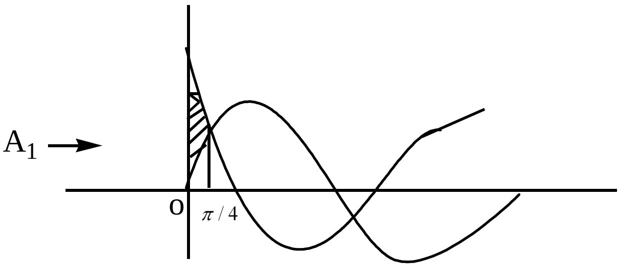
1) $2A_1=A_2$ and $A_1+A_2=1+\sqrt{2}$ **2)** $A_1:A_2=1:2$ and $A_1+A_2=1$

3) $A_1:A_2=1:\sqrt{2}$ and $A_1+A_2=1$ **4)** $A_1=A_2$ and $A_1+A_2=\sqrt{2}$

Key:3

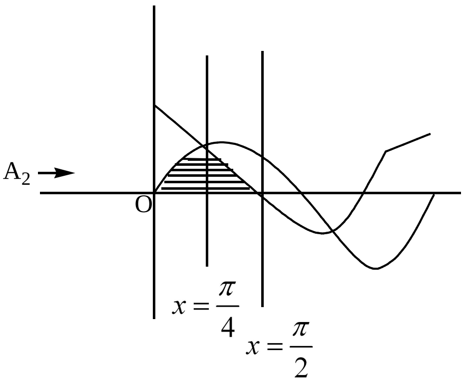
Solution:

$y=\sin x, y=\cos x, y=\text{axis}$



$$A_1=\int_0^{\pi/4}(\cos x-\sin x)dx=(\sin x+\cos x)_0^{\pi/4}=\sqrt{2}-1$$

$y=\sin x, y=\cos x, x-axis, x=\pi/2$



$$A_2=\int_0^{\pi/4}\sin x\,dx+\int_{\pi/4}^{\pi/2}\cos x=-(\cos x)_0^{\pi/4}+(\sin x)_{\pi/4}^{\pi/2}$$

$$=1-\frac{1}{\sqrt{2}}+1-\frac{1}{\sqrt{2}}=2-\sqrt{2}$$

$$A_1:A_2=\sqrt{2}-1:\sqrt{2}\left(\sqrt{2}-1\right)=1:\sqrt{2}$$

$$A_1+A_2=1$$

75. If the locus of the mid-point of the line segment from the point (3,2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to

1) $\frac{1}{4}$ **2)** $\frac{1}{3}$ **3)** $\frac{1}{2}$ **4)** 1

Key:3

Solution:

Point on circle = $(\cos\theta, \sin\theta) = p$

Let $Q = (3,2)$

Mid point of $PQ = \left(\frac{\cos\theta + 3}{2}, \frac{\sin\theta + 2}{2}\right)$

$= (x, y)$ $\Rightarrow (2x-3)^2 + (2y-2)^2 = 1$ $\Rightarrow \left(x - \frac{3}{2}\right)^2 + (y-1)^2 = \frac{1}{4}$

Radius $= \frac{1}{2}$

76. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :

1) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$ **2)** $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

3) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$ **4)** $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

Key:3

Solution:

$\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ Let $2n+1 = \lambda$ $\Rightarrow n = \frac{\lambda - 1}{2}$

$\frac{n^2 + 6n + 10}{(2n+1)!} = \frac{\left(\frac{\lambda-1}{2}\right)^2 + 6\left(\frac{\lambda-1}{2}\right) + 10}{\lambda!}$ $= \frac{\lambda^2 + 10\lambda + 29}{4(\lambda!)}$

Also $\lambda = 3, 5, 7, 9, 11, ...$

$\therefore$ Required $= \frac{1}{4}\sum \frac{\lambda(\lambda-1) + 11\lambda + 29}{\lambda!}$

$\lambda = 3, 5, 7, ...$

$$=\frac{1}{4}\left(\sum_{\lambda=3,5,7,.......}\left(\frac{1}{(\lambda-2)!}+\frac{11}{(\lambda-1)!}+\frac{29}{\lambda!}\right)\right)$$

$$=\frac{1}{4}\left(\left(\frac{1}{\lfloor 4}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 5}+.....\right)+11\left(\frac{1}{\lfloor 2}+\frac{1}{\lfloor 4}+.....+\right)+29\left(\frac{1}{\lfloor 3}+\frac{1}{\lfloor 5}+\frac{1}{\lfloor 7}+......\right)\right)$$

$$=\frac{1}{4}\left(\frac{e-\frac{1}{e}}{2}\right)+11\left(\frac{e+\frac{1}{e}-2}{2}\right)+29\left(\frac{e-\frac{1}{e}-2}{2}\right) \qquad =\frac{1}{8}\left(41e-\frac{19}{c}-80\right)$$

77. A seven digit number is formed using digits 3,3,4,4,4,5,5. The probability, that number so formed is divisible by 2 is

1) $\frac{4}{7}$ **2)** $\frac{1}{7}$ **3)** $\frac{6}{7}$ **4)** $\frac{3}{7}$

Key:4

Solution:

No. Of 7 digits number using 3, 3, 4, 4, 5, 5 $=\frac{\lfloor 7}{\lfloor 2\lfloor 3\lfloor 2}$

To get divisible by '2',

Digit in units place must be '4' $=\frac{\lfloor 6}{\lfloor 2\lfloor 2\lfloor 6 2}$

Required probability $=\frac{\frac{16}{\lfloor 2\lfloor 2\lfloor 2}}{\frac{\lfloor 7}{\lfloor 2\lfloor 3\lfloor 2}}=\frac{3}{7}$

78. Let $A=\{1,2,3,....10\}$ and $f:A\to A$ be defined as

$f(k)=\begin{cases} k+1 & if\ k\ is\ odd \\ k & if\ k\ is\ even \end{cases}$ then the number of possible functions $g:A\to A$ such that $gof=f$ is

1) 10_{C_5} **2)** 5^5 **3)** 10^5 **4)** $5!$

Key:3

Solution:

Range of $f=\{2,4,6,8,10\}$

$\therefore g(\lambda)=\lambda$ when 'λ' is even

$g: A \to A, A=\{1,2,3,.......,10\}$

No. of functions $=10^5$

79. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is

1) An equilateral triangle of height $\frac{2r}{3}$

2) An isosceles triangle with base equal to $2r$

3) A right angle triangle having two of its sides of length 2r and r

4) An equilateral triangle having each of its side of length $\sqrt{3}r$

Key:4

Solution:

Circum radius of $\Delta le = r$

Area of $\Delta^{le} = 2r^2 \sin A \sin B \sin C$

Maximum when $A=B=C=\pi/3$ $\Rightarrow$ equilateral

side $=2r\sin \pi/3 = \sqrt{3}r$

80. Let $f(x)$ be a differentiable function at $x=a$ with $f'(a)=2$ and $f(a)=4$. Then

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$$

1) $2a+4$ **2)** $2a-4$ **3)** $a+4$ **4)** $4-2a$

Key:4

Solution:

$$\underset{x\to a}{Lt}\frac{x.f(a)-a.f(x)}{x-a}$$

$$=\underset{x\to a}{Lt}\frac{f(a)-a.f^{'}(x)}{1}$$ (Using L' Hospital's rule)

$$=f(a)-af^{'}(a)$$

$$=4-2a$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is_____

Key:1000

Solution:

No. of 4 digits numbers = 9000

for g.cd of any number with 18 is 3

Number should be add, number divisible by 3, but not divisible by 9

Odd number divisible by 3 =1005,1011,------9999

These are $\rightarrow 1499$

no of 4 digit odd divisible by

=1017,1035,----------9999$\rightarrow$499 required =1499 – 499 =1000

82. Let the normals at all the points on given curve pass through a fixed point (a,b). If the curve passes through $(3,-3)$and$\left(4,-2\sqrt{2}\right)$, and given that $a-2\sqrt{2}b=3$, then $\left(a^2+b^2+ab\right)$is equal to ____

Key:9

Solution: equation of normal is $Y-y=\frac{-1}{m}(X-x)$

It passes through (a,b) $\Rightarrow b-y=\frac{-1}{m}(a-x)$

also passes through $(3,-3),(4,-2\sqrt{2})$

$\Rightarrow (b-y)dy=(x-a)dx \qquad \Rightarrow by-\frac{y^2}{2}=\frac{x^2}{2}-ax+c$

$\Rightarrow -3b-\frac{9}{2}=\frac{9}{2}-3a+c \Rightarrow 3a-3b-c=9 \rightarrow (1)$

$4a-2\sqrt{2}b-c=12 \rightarrow (2)$

$$\left.\begin{aligned}(1)-(2)\Rightarrow a+\left(3-2\sqrt{2}\right)b=3\\ a-2\sqrt{2}b=3\end{aligned}\right\}\text{solving b=0,a=3}$$

$\therefore a^2+b^2+ab=9$

83. If $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx,$ for $m,n\ge 1$, and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha I_{m,n,\alpha\in R}$, then α equals_____

Key:1

Solution:

$$\mathrm{I}_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx \qquad \text{Let } x=\frac{1}{x+1}$$

$$\mathrm{I}_{m,n}=\int_\infty^0 \frac{-t^{n-1}}{(t+1)^{m+n}}dt=\int_0^\infty \frac{t^{n-1}}{\left(t+1\right)^{m+n}}dt \to (1)$$

Similarly $I_{m,n}=\int_0^1 (1-x)^{m-1}x^{n-1}dx$

But $x=\frac{1}{y+1}$ $\quad I_{m,n}=\int_0^\infty \frac{y^{m-1}}{(y+1)^{m+n}}dy \to (2)$

$$(1)+(2)\Rightarrow 2I_{m,n}=\int_0^\infty \frac{t^{n-1}}{(t+1)^{m+n}}dt+\int_0^\infty \frac{y^{m-1}}{(y+1)^{m+n}}dy$$

$$=\int_0^\infty \frac{x^{n-1}+x^{m-1}}{(x+1)^{m+n}}dx \quad =\int_0^1 \frac{x^{n-1}+x^{m-1}}{(x+1)^{m+n}}dx+\int_1^\infty \frac{x^{n-1}+x^{m-1}}{(x+1)^{m+n}}dx$$

Put $x=\frac{1}{z}=\int_0^1 \frac{x^{n-1}+x^{m-1}}{(x+1)^{m+n}}dx+\int_0^1 \frac{x^{n-1}+x^{m-1}}{(x+1)^{m+n}}dx \; 2\mathrm{I}_{m,n}=2\int_0^1 \frac{x^{n-1}+x^{m-1}}{(x+1)^{m+n}}dx$

$$\Rightarrow I_{m,n}=\int_0^1 \frac{x^{n-1}+x^{m-1}}{\left(x+1\right)^{m+n}}dx$$

84. Let α and β be two real numbers such that $\alpha+\beta=1$ and $\alpha\beta=-1$ Let $p_n=(\alpha)^n+(\beta)^n$, $P_{n-1}=11$ and $P_{n+1}=29$ for some integer $n\geq 1$. Then, the value of p_n^2 is ______

Key:324

Solution:

$p_n=\alpha^n+\beta^n \qquad p_{n-1}=11, p_{n+1}=29 \qquad \alpha+\beta=1, \alpha\beta=-1$

The Question Equation having roots α,β is $x^2-x-1=0$

$\Rightarrow \alpha^2=\alpha+1, \beta^2=\beta+1 \qquad \Rightarrow \alpha^n=\alpha^{n-1}+\alpha^{n-2}, \beta^n=\beta^{n-1}+\beta^{n-2}$

$\Rightarrow \alpha^n=\alpha^{n-1}+\alpha^{n-2}, \beta^n=\beta^{n-1}+\beta^{n-2} \; \Rightarrow p \sim= p_{n-1}+p_{n-2}$

$\Rightarrow p_{n+1}=p_n+p_{n-1} \Rightarrow p_n=29-11=18 \qquad \therefore p_n^2=324$

85. If the matrix $A=\begin{bmatrix}1&0&0\\0&2&0\\3&0&-1\end{bmatrix}$ satisfies the equation $A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1&0&0\\0&4&0\\0&0&1\end{bmatrix}$ for some real numbers α and β then $\beta-\alpha$ is equal to ___

Key:4

Solution:

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow A^4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$symilrly\ A^{19} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{pmatrix} A^{20} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore A^{20} + \alpha A^{19} + gbA = \begin{pmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \alpha+\beta = 0\ 2^{20} + \alpha\left(2^{19}\right) + 2\beta = 4$$

$$\Rightarrow \alpha = -2 \Rightarrow \beta - \alpha = 4$$

86. Let $X_1, X_2, ..., X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i - \alpha) = 36$ and $\sum_{i=1}^{18}(X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ________

Key:4

Solution:

$$(\Sigma x_i) - 18\alpha = 36$$

$$\Sigma x_i = 18(\alpha + 2)$$

$$\left(\Sigma x_i^2\right) + 18\beta^2 - 2\beta\Sigma x_i = 90$$

$$\Rightarrow \Sigma x_o^2 = 90 - 18\beta^2 + 36(\alpha+2)\beta$$

$$\sigma^2 = 1 \Rightarrow \frac{1}{18}\left(\Sigma x_i^2\right) - \left(\frac{\Sigma x_i}{18}\right) = 1$$

$$\Rightarrow \frac{1}{18}\left(90-18\beta^2+36\alpha\beta+72\beta\right) \qquad -(\alpha+2)^2=1$$

$$\Rightarrow 5-\beta^2+2\alpha\beta+4\beta-\alpha^2-4-4\alpha=1$$

$$\Rightarrow \alpha^2+\beta^2-2\alpha\beta+4(\alpha-\beta)=0$$

$$\Rightarrow (\alpha-\beta)(\alpha-\beta+4)=0$$

$$\Rightarrow \alpha-\beta=-4(\therefore \alpha\neq\beta)$$

$$\therefore |\alpha-\beta|=4$$

87. If the arithmetic mean and geometric mean of the pth and qth terms of the sequence $-16,8,-4,2,.....$ satisfy the equation $4x^2-9x+5=0$, then $p+q$ is equal to _______.

Key:10

Solution:

Given sequence is $-16,8,-4,2,..........$

$$=-16,-16\left(\frac{-1}{2}\right),-16\left(\frac{-1}{2}\right)^2,-16\left(\frac{-1}{2}\right)^3,....$$

$$t_p=-16\left(\frac{-1}{2}\right)^{p-1},t_q=-16\left(\frac{-1}{2}\right)^{q-1}$$

$\frac{t_p+t_q}{2},\sqrt{t_pt_q}$ are roots of $4x^2-9x+5=0$

Which are $\frac{5}{4},1$

$$\sqrt{t_pt_q}=1\Rightarrow t_pq_q=1$$

$$\Rightarrow 2^8\left(\frac{-1}{2}\right)^{p+q-2}=1$$

$\Rightarrow p+q-2$ is even, $10-(p+q)=0$

$$\Rightarrow p+q=10$$

88. Let L be a common tangent line to the curves $4x^2+9y^2=36$ and $(2x)^2+(2y)^2=31$. Then the square of the slope of the line is ________.

Key:3

Solution:

Let m be the slope of tangent to $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\Rightarrow$ ‘L’ is $y=mx\pm\sqrt{9m^2+4}$

It is a tangent to circle $x^2+y^2=\frac{31}{4}$

$$\Rightarrow \frac{\sqrt{9m^2+4}}{\sqrt{m^2+1}} = \sqrt{\frac{31}{4}}$$

$$\Rightarrow 4\left(9m^2+4\right) = 31m^2+31$$

$$\Rightarrow 5m^2 = 15 \Rightarrow m^2 = 3$$

89. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4+10x^3+10x^2+10x+10$ lie in the interval $(a, a+1)$. Then, $|a|$ is equal to _______.

Key:2

Solution:

$$f(x) = 2x^5+5x^4+10x^3+10x^2+10x+10$$

$$\Rightarrow f'(x) = 10x^4+20x^3+30x^2+20x+10$$

$$= 10\left[x^4+2x^3+3x^2+2x+1\right]$$

$$= 10x^2\left(x^2+\frac{1}{x^2}+2\left(x+\frac{1}{x}\right)+3\right)$$

$$= 10x^2\left(\left(x+\frac{1}{x}\right)^2-2+2\left(x+\frac{1}{x}\right)+3\right)$$

$$= 10x^2\left(\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)+1\right)$$

$$= 10x^2\left(\left(x+\frac{1}{x}\right)+1\right)^2$$

$> 0 \ \forall x \in R - \{0\}$

$\exists$ exactly one real root.

$f(-1) = 3, f(-2) = -34$

$f(-1).f(-2) < 0 \Rightarrow 7$ a root in $(-2, -1)$

$\therefore a = -2 \Rightarrow |a| = 2$

90. Let z be those complex numbers which satisfy $|z+5| \le 4$ and $z(1+i)+\overline{z}(1-i) \ge -10, i = \sqrt{-1}$.

If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is ________.

Key:48

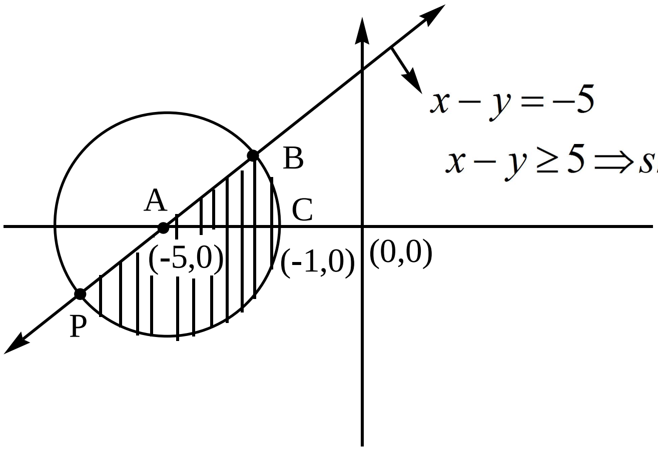
Solution:

$|z+5| \le 4, \qquad z(1+i)+\overline{z}(1-i) \ge -10$

$|z+5| \le 4 \Rightarrow$ interior part and circumference of circle with centre (-5, 0) and radius = 4

$z(1+i)+\bar{z}(1-i)\geq -10 \Rightarrow 2(x-y)\geq -10$

$\Rightarrow x-y\geq -5$



For max of $|z+1|^2$

$\therefore\ p=\left(-5,-4\left(\frac{1}{\sqrt{2}}\right),0-4\left(\frac{1}{\sqrt{2}}\right)\right)$

$=\left(-5-2\sqrt{2},-2\sqrt{2}\right)$

$|z+1|^2_{\max}=(CP)^2$

$=\left(-4-2\sqrt{2}\right)^2+\left(-2\sqrt{2}\right)^2$

$=32+16\sqrt{2}$

$=\alpha+\beta\sqrt{2}$

$\alpha=32,\beta=16$